Exam

Name________________________________

1) Let a curve be defined by $x = 2t^2$, $y = t^3 - 3t$. Then find the point(s) (x, y) on that curve where **the tangent line is horizontal** ? 1) ______

A) $(-2, 1)$ and $(2, -1)$
B) $(2, 1)$ and $(0, 0)$
C) $(4, 2)$ and $(0, -3)$
D) $(4, 1)$ and $(4, 2)$
E) $(2, -2)$ and $(2, 2)$

2) Set up the integral to find **the area** under the cycloid defined by the equations $x = t - \sin(t)$, $y = 1 - \cos(t)$, $0 < t < 2\pi$. 2) ______

A) $\int_0^{2\pi} (1 - \sin(t))^2 \, dt$

B) $\int_0^{2\pi} \sin(t) \cos(t) \, dt$

C) $\int_0^{2\pi} \cos^2(t) \, dt$

D) $\int_0^{2\pi} \sin^2(t) \, dt$

E) $\int_0^{2\pi} (1 - \cos(t))^2 \, dt$

3) Which one is **true** for the following integral : $\int_1^{\infty} \frac{1}{x^{2/3}}\, dx$? 3) ______

A) It is equal to $\frac{2}{3}$

B) It is equal to $\frac{1}{3}$

C) Divergent

D) Type 2

E) It is equal to $\frac{5}{3}$

4) Calculate **the arclength** of the curve defined by the equations $x = \sin(3t)$, $y = \cos(3t)$, $0 < t < \pi$. 4) ______

A) π^2 B) 2π C) 9π D) 6π E) 3π

5) Let $y = \ln(\cos x)$, $0 < x < \frac{\pi}{2}$. Set up the integral (in most simplified form) to find the **arc lenght** of y. 5) ______

A) $\int_0^{\pi/2} \sec x \; dx$

B) $\int_0^{\pi/2} \sin x \; dx$

C) $\int_0^{\pi/2} \ln(\cos x) \; dx$

D) $\int_0^{\pi/2} \cos x \; dx$

E) $\int_0^{\pi/2} \tan x \; dx$

6) Let $y = A$, where A is a constant . If the **surface area** obtained rotating y around x axis over $[0, 3]$ is 48π, then A is 6) ______

A) 144 B) 16 C) 45 D) 8 E) 3

7) Which one is **true** for the following integral : $\int_2^{\infty} e^{-5x}\,dx$? 7) ______

A) Divergent

B) It is equal to $\frac{1}{5\,e^{10}}$

C) Neither Type 1 nor Type 2

D) Type 2

E) It is equal to $2\,\frac{1}{5e}$

8) Let $\frac{x^2 - x + 2}{x\left(x^2 + 4\right)} = \frac{A}{x} + \frac{Bx + C}{\left(x^2 + 4\right)}$, $A = B = \frac{1}{2}$, $C = -1$. Then calculate 8) ______

$\int \frac{x^2 - x + 2}{x\left(x^2 + 4\right)}\,dx.$

A) $\frac{1}{2}\ln(x) + \frac{1}{4}\ln\left(x^2 + 4\right) - \frac{x}{2} + C$

B) $2\ln(x) + 4\ln\left(x^2 + 4\right) - 2\arctan\left(\frac{x}{2}\right) + C$

C) $\frac{1}{2}\ln(x) + \frac{1}{4}\ln\left(x^2 + 4\right) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

D) $\frac{1}{2}\ln(x) + \frac{1}{4}\left(x^2 + 4\right) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

E) $\frac{1}{2}x + \frac{1}{4}\ln\left(x^2 + 4\right) - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$

9) Which one of the following is **false.** 9) ______

A) Suppose that a circle rolls along a horizontal line without slipping. As the circles rolls along the line, a point P on the circle traces out a curve called a cycloid.

B) The inverted cycloid is the solution to the Brachistochrome problem.

C) The parametric equations of a cycloid are $x = a(t - \sin t)$ and $y = a(1 - \cos t)$

D) The inverted cycloid is the solution to the Tautochrone problem.

E) The curve reprersented by $x = \cos t$, $y = \sin t$, $0 < t < 2\pi$ is a parabola passing through (1, 0).

10) Which one of the following is **false** ? 10) ______

A) $\int_0^3 \frac{1}{x - 3}\, dx$ is a Type 2 Improper Integral.

B) The integrand may fail to be defined, or fail to be continuous, at a point in the interval of integration, typically an endpoint. This leads to what is sometimes called **a Type 2** improper Integral.

C) We may, for some reason, want to define an integral on an interval extending to $\pm\infty$. This leads to what is sometimes called **a Type 1** Improper Integral.

D) $\int_2^\infty \frac{1}{x - 1}\, dx$ is a Type 1 Improper Integral.

E) $\int_2^{13} \frac{1}{x - 3}\, dx$ is neither Type 1 nor Type 2

11) Calculate $\lim\limits_{x \to 0} \frac{1 - \cos(x)}{x^2}$ 11) ______

A) 0 B) 1 C) 2 D) $\frac{1}{2}$ E) ∞

12) According to the method of partial fractions, there is an equation of the form : 12) ______

$$\frac{x^3 + 3x + 1}{(x+1)^2 (x-2)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-2)} + \frac{D}{(x-2)^2}$$

for some numbers A, B , C, D. Then D is

A) $\frac{2}{3}$ B) 2 C) $\frac{5}{3}$ D) $\frac{1}{3}$ E) $\frac{4}{3}$

13) Let $y = 2$, $a < x < b$. If the **arc lenght** of y is 15, then b - a is 13) ______

A) 30 B) $\frac{15}{2}$ C) 1 D) 15 E) 2

14) Set up the partial fraction decomposition : $\frac{5x^3 - 21x^2 + 7x - 1}{(x-3)^3 (x^2+8)^2}$: 14) ______

A) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{Dx+E}{(x^2+8)^2}$

B) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{Dx^2+Ex+F}{x^2+8} + \frac{Gx^2+Hx+K}{(x^2+8)^2}$

C) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{Dx+E}{x^2+8} + \frac{Fx+G}{(x^2+8)^2}$

D) $\frac{A}{x-3} + \frac{Bx}{(x-3)^2} + \frac{Cx+H}{(x-3)^3} + \frac{Dx+E}{x^2+8} + \frac{Fx+G}{(x^2+8)^2}$

E) $\frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3} + \frac{D}{x^2+8} + \frac{E}{(x^2+8)^2}$

15) Which one is **true** for $F(x) = \frac{3x^3 + 5x^2 - 11x + 3}{x + 3}$. 15) ______

A) It is equal to $3x^2 - 4x + 1$

B) It is equal to $3x^2 - 4x + 1 + \frac{x - 1}{x + 3}$

C) It is equal to $3x^2 - 4x + 1 + \frac{2}{x + 3}$

D) It is equal to $3x^2 - 4x + 1 + \frac{4x - 3}{x + 3}$

E) It is a proper rational function